2 Theory & Definitions

Exercise 1. Let P, Q be convex polyhedra and let P_I, Q_I be the associated integer polyhedra, i.e. $P_I = \text{conv}(P \cap \mathbb{Z}^n)$. Decide, whether the following inclusions hold (prove or give a counterexample):

a) $(P+Q)_I \subseteq P_I + Q_I$, [2 pts]

b) $(P+Q)_I \supseteq P_I + Q_I$. [2 pts]

Exercise 2. Is the following matrix totally unimodular?

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

[1 pt]

Exercise 3. Let $A \in \mathbb{Z}^{n \times n}$ be a unimodular matrix. Is the inverse A^{-1} also unimodular? Is A^{-1} totally unimodular for a non-singular totally unimodular matrix A? [2 pts]

Exercise 4. Let $\sigma(\cdot)$ denote the size of the binary representation of a given number.

a) Show that $\sigma(a+b) \leq \sigma(a) + \sigma(b)$ holds for each $a, b \in \mathbb{Z}$. [1 pt]

b) Show that $\sigma(a+b) \leq \sigma(a) + \sigma(b)$ does not hold, in general, for $a, b \in \mathbb{Q}$. [1 pt]

c) Show that $\sigma(a \cdot b) \leq \sigma(a) + \sigma(b)$ holds for each $a, b \in \mathbb{Q}$. [1 pt]