

3 Methods for Solving Integer Programs

Exercise 1. Solve the integer linear program

$$\begin{aligned} \max \quad & x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + 5x_2 \leq 12 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{aligned}$$

- a) using Gomory's cutting plane method,
b) using branch-and-bound with linear programming relaxations. [3 pts]

Exercise 2. Let $\alpha > 0$ be given. Find the minimal description of the convex hull of the set

$$M = \{(x, y) \in \mathbb{Z} \times \mathbb{R} : x - y \leq \alpha, y \geq 0\}. \quad [2 \text{ pts}]$$

Exercise 3. Show that a cutting plane for a pure integer program can also be defined as follows: Let $d_{k0} \notin \mathbb{Z}$, i.e. the right-hand side of the k^{th} row is not integer in the final ℓ -table of the lexicographic dual simplex method. Then, the cutting plane is given by

$$\sum_{j \in I} x_{N_j} = 1, \text{ where } I = \{j \in \{1, \dots, n\} : d_{kj} \notin \mathbb{Z}\}. \quad [3 \text{ pts}]$$

Exercise 4. Modify the branch-and-bound algorithm to find a "sufficiently good" feasible solution whose objective value is within $p\%$ of the optimum value. [2 pts]