5 Special Cases

Exercise 1. Decide which of the following variants of the knapsack problem with $A, a, b, c \geq 0$ can be transformed into one of the other forms $(P1) - (P4)$:

(P1) max $c^T x$ subject to $a^T x \leq b$, $x \in \{0,1\}^n$,

(P2) max $c^T x$ subject to $a^T x \leq b$, $x \geq 0$, $x \in \mathbb{Z}^n$,

(P3) max $c^T x$ subject to $A^T x \leq b$, $x \in \{0,1\}^n$,

(P4) max $c^T x$ subject to $a^T x = b$, $x \in \{0,1\}^n$. [2 pts]

Exercise 2. Find all facet-defining inequalities for the symmetric TSP with $n = 4$. [3 pts]

Exercise 3. Let two integers $n \geq 3$ and $m \geq 2$ be given. Let P denote the convex hull of the points $(x, y) \in \{0,1\}^{m \times n} \times \{0,1\}^n$ satisfying

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = 1, && i = 1, \ldots, m, \\ &0 \leq x_{ij} \leq y_j \leq 1, && i = 1, \ldots, m,\ j = 1, \ldots, n. \end{aligned}$$

a) Show that $y_j \leq 1$ defines a facet of P for all $j = 1, \ldots, n$.

b) Show that $x_{ij} \leq y_j$ defines a facet of P for all $i = 1, \ldots, m$ and $j = 1, \ldots, n$. [2 pts]

Exercise 4. Let x^* denote the optimal solution of a set covering problem and let x^H be the heuristic solution found by the greedy algorithm. Find an instance, for which the approximation bound

$$c^T x^H \leq H(d) c^T x^*,$$

with $d = \max_{j=1,\ldots,n} |S_j|$ and $H(k) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}$, is achieved as tightly as possible. [3 pts]

Bonus

Exercises from this section can be used to replace any missing points, including homework set 5.

Exercise 1. Find an interesting application for one of the special-case problems discussed in the lecture (knapsack, TSP, facility location, set cover). Write down a (possibly simplified) model for the chosen application and explain the necessary constraints and variables. [1 pt]

Exercise 2. Suggest a new heuristic, cutting plane or branching rule for one of the special-case problems and discuss its properties. [1 pt]