Exercise 1. Prove NP-hardness of integer linear programming by showing a reduction from the following NP-hard problems:

a) uncapacitated facility location problem,

b) (symmetric) travelling salesman problem.

Exercise 2. Using the lexicographic method, find optimum of the linear program:

$$\begin{array}{ll} \max & x_1 + 2x_2 \\ \text{s.t.} & -x_1 + x_2 \leq 1 \\ & 3x_1 - 4x_2 \leq 6 \\ & x_1 + x_2 \leq 4 \\ & x_1 + 3x_2 \leq 9 \\ & x_2 \leq \frac{5}{2} \\ & x_1, x_2 \geq 0 \end{array}$$

Exercise 3. Given a set X and a point x^*, find a valid inequality for X cutting off the point x^*.

a) $X = \{(x_1, x_2) \in \mathbb{Z} \times \{0,1\} : 2x_1 - x_2 \leq 2\}$ and $x^* = (1.5, 1)$,

b) $X = \{(x_1, x_2) \in \mathbb{R}_+ \times \mathbb{Z}_+ : x_1 \leq 9, x_1 \leq 4x_2\}$ and $x^* = (9, \frac{9}{4})$.

Exercise 4. Formulate an integer linear program for the 0-1 knapsack problem: Given a set of n items, each with a (positive) weight w_i and a (positive) value v_i, choose items to pack into a knapsack of a weight capacity W to get the maximum total value. Show that the linear relaxation of the problem has an optimal solution with at most one fractional entry.

Exercise 5. Solve the following integer linear programs using the first Gomory algorithm:

$$\begin{array}{ll} \max & x_1 + x_2 \\ \text{s.t.} & 3x_1 + 6x_2 \leq 10 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array} \tag{a}$$

$$\begin{array}{ll} \max & x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 7 \\ & -3x_1 + x_2 \leq -1 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array} \tag{b}$$

$$\begin{array}{ll} \max & x_1 - x_2 \\ \text{s.t.} & -\frac{1}{3}x_1 + x_2 \leq \frac{1}{3} \\ & x_1 - \frac{1}{3}x_2 \leq \frac{1}{3} \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array} \tag{c}$$

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