Exercise 1.   Solve the following pure integer linear programs using the first Gomory algorithm:

$$
\begin{aligned}
\max \quad & x_2 \\
\text{s.t.} \quad & 2x_1 + x_2 \le 7 \\
& -3x_1 + x_2 \le -1 \\
& x_1, x_2 \ge 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
\tag{a}
$$

$$
\begin{aligned}
\max \quad & x_1 - x_2 \\
\text{s.t.} \quad & -\tfrac{1}{3}x_1 + x_2 \le \tfrac{1}{3} \\
& x_1 - \tfrac{1}{3}x_2 \le \tfrac{1}{3} \\
& x_1, x_2 \ge 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
\tag{b}
$$

Exercise 2.   Solve the following integer linear programs using the second Gomory algorithm:

$$
\begin{aligned}
\max \quad & -x_1 + x_2 \\
\text{s.t.} \quad & x_2 \le 9 \\
& -4x_1 + x_2 \le 0 \\
& x_1, x_2 \ge 0 \\
& x_1 \in \mathbb{Z}, x_2 \in \mathbb{R}
\end{aligned}
\tag{a}
$$

$$
\begin{aligned}
\max \quad & x_1 - x_2 \\
\text{s.t.} \quad & -\tfrac{1}{3}x_1 + x_2 \le \tfrac{1}{3} \\
& x_1 - \tfrac{1}{3}x_2 \le \tfrac{1}{3} \\
& x_1, x_2 \ge 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
\tag{b}
$$

Exercise 3.   How can we detect unbounded linear programs when using the lexicographic simplex method? Try to solve the following program:

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_2 \le 2 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

Exercise 4.   How can we detect an infeasible linear relaxation, or an infeasible integer program when using the Gomory algorithms?

Exercise 1. Solve the following pure integer linear programs using the first Gomory algorithm:

$$
\begin{aligned}
\max \quad & x_2 \\
\text{s.t.} \quad & 2x_1 + x_2 \leq 7 \\
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& x_1, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned} \tag{a}
$$

$$
\begin{aligned}
\max \quad & x_1 - x_2 \\
\text{s.t.} \quad & -\tfrac{1}{3}x_1 + x_2 \leq \tfrac{1}{3} \\
& x_1 - \tfrac{1}{3}x_2 \leq \tfrac{1}{3} \\
& x_1, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned} \tag{b}
$$

Exercise 2. Solve the following integer linear programs using the second Gomory algorithm:

$$
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\end{aligned} \tag{a}
$$

$$
\begin{aligned}
\max \quad & x_1 - x_2 \\
\text{s.t.} \quad & -\tfrac{1}{3}x_1 + x_2 \leq \tfrac{1}{3} \\
& x_1 - \tfrac{1}{3}x_2 \leq \tfrac{1}{3} \\
& x_1, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned} \tag{b}
$$

Exercise 3. How can we detect unbounded linear programs when using the lexicographic simplex method? Try to solve the following program:

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_2 \leq 2 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Exercise 4. How can we detect an infeasible linear relaxation, or an infeasible integer program when using the Gomory algorithms?