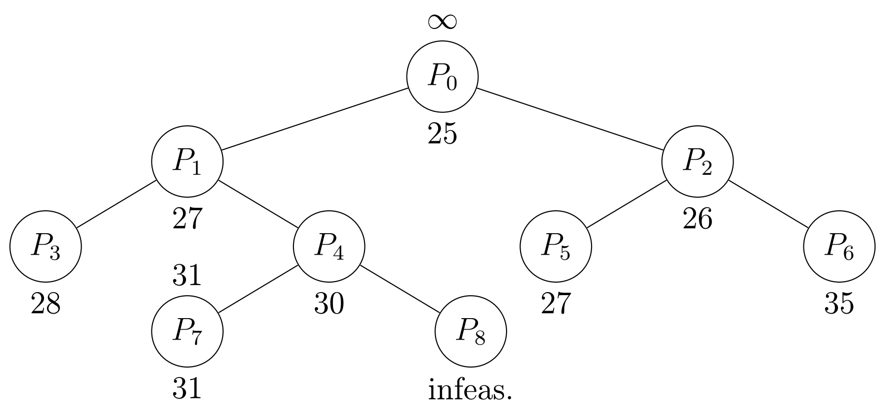
Exercise 1. Find a Chvátal-Gomory cut for the set

$$\{x \in \mathbb{Z}^5 : 9x_1 + 12x_2 + 8x_3 + 17x_4 + 13x_5 \geq 50, x \geq 0\}$$

cutting off the point $(0, \frac{25}{6}, 0, 0, 0)$.

Exercise 2. Consider the following branch-and-bound tree for a minimization problem:



Give the tightest possible bounds on the optimal value. Which nodes can be pruned and which must be explored further?

Exercise 3. Solve the following integer problems by branch-and-bound:

$$\begin{aligned} \max \quad & 17x_1 + 10x_2 + 25x_3 + 17x_4 \\ \text{s.t.} \quad & 5x_1 + 3x_2 + 8x_3 + 7x_4 \leq 12 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned} \tag{a}$$

$$\begin{aligned} \max \quad & 13x_1 + 8x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 10 \\ & 5x_1 + 2x_2 \leq 20 \\ & x_1, x_2 \in \mathbb{N}_0 \end{aligned} \tag{b}$$

Exercise 4. Consider the integer programming problem:

$$\begin{aligned} \min \quad & x_{n+1} \\ \text{s.t.} \quad & 2x_1 + 2x_2 + \ldots + 2x_n + x_{n+1} = n \\ & x_1, x_2, \ldots, x_{n+1} \in \{0, 1\} \end{aligned}$$

Further, let us assume that n is odd.

a) What objective values can be attained by feasible integer solutions?

b) What is the optimal value of a linear relaxation in branch-and-bound, when no more than $\frac{n}{2}$ of the variables $x_1, \ldots, x_n$ are fixed to 0 or 1? Can the corresponding optimal solution be integer?

c) Argue that the branch-and-bound algorithm (with a linear relaxation and branching by a fractional variable) will require the enumeration of an exponential number of subproblems to solve the program.

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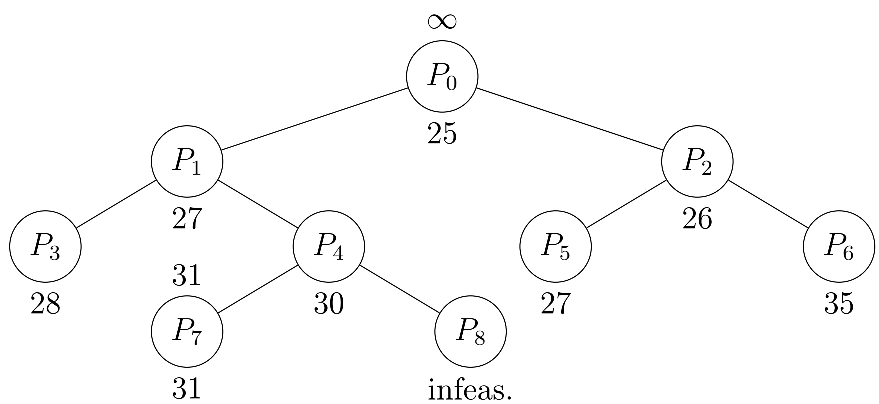

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