Exercise 1. Find the best bounds for each variable in the following system:

$$\begin{aligned} &2x_1 + 7x_2 - 3x_3 + 6x_4 - 9x_5 + \ x_6 \leq -12 \\ &\ x_1 - 2x_2 + \ x_3 + 4x_4 + 2x_5 - 3x_6 \leq 13 \\ &x_1 \in [1,4],\ x_2 \in [0,7],\ x_3 \in [4,10],\ x_4 \geq 2,\ x_5 \in [0,2],\ x_6 \geq 0 \\ &x_1, \ldots, x_6 \in \mathbb{Z} \end{aligned}$$

Exercise 2. Consider the set $X = \{x \in \{0,1\}^n : \sum_{j=1}^n a_j x_j \leq b\}$ with $a_j \geq 0$ for each $j \in \{1, \ldots, n\}$. Under what conditions

a) is the set X empty?

b) is the constraint $\sum_{j=1}^n a_j x_j \leq b$ redundant?

c) is the constraint $x_j = 0$ valid?

d) is the constraint $x_i + x_j \leq 1$ valid?

Exercise 3. Solve the following 0-1 program by deriving and combining logical inequalities:

$$\begin{aligned} 7x_1 + 3x_2 - 4x_3 - 2x_4 &\leq 1 \\ -2x_1 + 7x_2 + 3x_3 + \ x_4 &\leq 6 \\ - 2x_2 - 3x_3 - 6x_4 &\leq -5 \\ 3x_1 \qquad - 2x_3 \qquad\ \ &\geq -1 \\ x_1, \ldots, x_4 &\in \{0,1\} \end{aligned}$$

Exercise 4. Use preprocessing techniques to simplify the integer linear program:

$$\begin{aligned} \max \quad & 2x_1 + \ x_2 - \ x_3 \\ \text{s.t.} \quad & 5x_1 - 2x_2 + 8x_3 \leq 15 \\ & 8x_1 + 3x_2 - \ x_3 \geq 9 \\ & \ x_1 + \ x_2 + \ x_3 \leq 6 \\ & x_1 \in [0,3],\ x_2 \in [0,1],\ x_3 \geq 1 \\ & x_1, x_2, x_3 \in \mathbb{Z} \end{aligned}$$

Exercise 5. Find an approximation of the optimal solution using the relax & fix heuristics:

$$\begin{aligned} \max \quad & 13x + 8y \\ \text{s.t.} \quad & x + 2y \leq 10 \\ & 5x + 2y \leq 20 \\ & x, y \in \mathbb{N}_0 \end{aligned}$$

Exercise 6. Solve the following knapsack problem using the pseudopolynomial algorithm:

$$\begin{aligned} \max \quad & x_1 + 5x_2 + 3x_3 + \ x_4 + 2x_5 \\ \text{s.t.} \quad & 3x_1 + 4x_2 + 3x_3 + 2x_4 + \ x_5 \leq 7 \\ & x_1, \ldots, x_5 \in \{0,1\} \end{aligned}$$

Exercise 7. Show that we can w.l.o.g. assume $a > 0, c > 0$ for a knapsack problem of type

$$\max\ c^T x \text{ s.t. } a^T x \leq b,\ x \geq 0,\ x \in \mathbb{Z}^n.$$

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