(4) Heuristics, decompositions & column generation

Exercise 4.1. Consider the set $X = \{x \in \{0,1\}^n : \sum_{j=1}^n a_j x_j \leq b\}$ with $a_j \geq 0$ for each $j \in \{1, \ldots, n\}$. Under what conditions

(a) is the set X empty?

(b) is the constraint $\sum_{j=1}^n a_j x_j \leq b$ redundant?

(c) is the constraint $x_j = 0$ valid?

(d) is the constraint $x_i + x_j \leq 1$ valid?

Exercise 4.2. Solve the following 0-1 program by deriving and combining logical inequalities:

$$\begin{aligned} 7x_1 + 3x_2 - 4x_3 - 2x_4 &\leq 1 \\ -2x_1 + 7x_2 + 3x_3 + x_4 &\leq 6 \\ -2x_2 - 3x_3 - 6x_4 &\leq -5 \\ 3x_1 - 2x_3 &\geq -1 \\ x_1, \ldots, x_4 &\in \{0,1\} \end{aligned}$$

Exercise 4.3. Apply Lagrangian relaxation with the subgradient method to the integer program:

$$\begin{aligned} \max \quad & 16x_1 + 10x_2 + 4x_4 \\ \text{s.t.} \quad & 8x_1 + 2x_2 + x_3 + 4x_4 \leq 10 \\ & x_1 + x_2 \leq 1 \\ & x_3 + x_4 \leq 1 \\ & x_1, x_2, x_3, x_4 \in \{0,1\} \end{aligned}$$

Exercise 4.4. Apply the column generation algorithm to the following cutting stock problem:

- company stores rolls of length 218 cm,
- a customer has ordered 44 pieces of length 81 cm, 3 pieces of length 70 cm and 48 pieces of length 68 cm.