(1) The Art of Formulating Integer Programs

Problem 1.1. Given binary variables $b_1, b_2, b_3 \in \{0, 1\}$ and continuous variables $x_1, x_2 \in [0, K]$ for some $K > 0$, linearize the following constraints:

(a) $b_3 = b_1 \cdot b_2$, [2 pts]

(b) $x_2 = b_1 \cdot x_1$. [2 pts]

Problem 1.2. Let two values K_1, K_2 be given, such that $0 < K_1 < K_2$. Formulate integer linear programming constraints representing the requirement

$$x \in \{0\} \cup [K_1, K_2],$$

where $x \in \mathbb{R}^+$ is a continuous variable. [2 pts]

Problem 1.3. Nurses at the St. Charles hospital work 8-hour shifts starting at 0:00, 4:00, 8:00, 12:00, 16:00 and 20:00. Find an integer linear programming model to determine the minimum number of nurses needed to satisfy the following requirements:

Time interval	Minimum number of nurses
00:00–04:00	3
04:00–08:00	8
08:00–12:00	10
12:00–16:00	12
16:00–20:00	14
20:00–00:00	8

[2 pts]

Problem 1.4. Charles wants to enroll in some of the classes at his university. Using integer linear programming constraints, help him model the following requirements:

(a) If Charles takes k or more classes from the set $\{p_1, \ldots, p_n\}$, then he also has to take classes q_1 and q_2. [3 pts]

(b) Charles can take class p_3 only if he also takes p_1 or p_2, but not both. [2 pts]

Problem 1.5. Let $\{x \in \mathbb{R}^n : Ax \leq b\}$ be a bounded polyhedron. Given some $x^* \in \mathbb{R}^n$, formulate the requirement

$$y \in \{x \in \mathbb{Z}^n : Ax \leq b\} \backslash \{x^*\}$$

using integer linear programming constraints. [3 pts]

Problem 1.6. Formulate an integer linear programming model for any logic puzzle of your choice other than Sudoku (you can consult the list of Nikoli[1] puzzles for inspiration). [2–4 pts]

[1] `https://en.wikipedia.org/wiki/Nikoli_(publisher)`