(2) Integer polyhedra, unimodularity & complexity

Problem 2.1. Let P, Q be convex polyhedra and let P_I, Q_I be the associated integer polyhedra, i.e. $P_I = \operatorname{conv}(P \cap \mathbb{Z}^n)$. Decide, whether the following inclusions hold (prove the claim or give a counterexample):

(a) $(P + Q)_I \subseteq P_I + Q_I$,

(b) $(P + Q)_I \supseteq P_I + Q_I$. [5 pts]

Problem 2.2. Is the following matrix totally unimodular?

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

[2 pts]

Problem 2.3. Show that for proving total unimodularity of a matrix, it is not sufficient to consider only submatrices formed by consecutive rows/columns.
How many square submatrices does a matrix $A \in \mathbb{R}^{n \times n}$ have? [3 pts]

Problem 2.4. Let $A \in \mathbb{Z}^{n \times n}$ be a non-singular matrix.

(a) If A is unimodular, is the inverse matrix A^{-1} also unimodular?

(b) If A is totally unimodular, is the inverse matrix A^{-1} also totally unimodular? [5 pts]

Problem 2.5. Let $\sigma(\cdot)$ denote the size of the binary representation of a given number.

(a) Show that $\sigma(a + b) \leq \sigma(a) + \sigma(b)$ does not hold, in general, for $a, b \in \mathbb{Q}$.

(b) Show that $\sigma(a \cdot b) \leq \sigma(a) + \sigma(b)$ holds for each $a, b \in \mathbb{Q}$. [5 pts]