(3) Methods: Cutting planes & Branch-and-bound

Problem 3.1. Formulate a valid inequality (with respect to the set X), which cuts the point x^*:

(a) for $X = \{(x_1, x_2) \in \mathbb{Z} \times \{0, 1\} : 2x_1 - x_2 \leq 2\}$ and the point $x^* = (1.5, 1)$,

(b) for $X = \{(x_1, x_2) \in \mathbb{R}_+ \times \mathbb{Z}_+ : x_1 \leq 9, x_1 \leq 4x_2\}$ and the point $x^* = (9, \frac{9}{4})$. [2 pts]

Problem 3.2. Solve the integer linear program

$$\begin{array}{ll} \max & x_1 + 3x_2 \\ \text{subject to} & x_1 + 5x_2 \leq 12 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{array}$$

(a) using (first) Gomory's cutting plane method,

(b) using branch-and-bound with linear programming relaxations. [7 pts]

Problem 3.3. Solve the mixed integer program using (second) Gomory's cutting plane method:

$$\begin{array}{ll} \max & -x_1 + x_2 \\ \text{subject to} & x_2 \leq 9, \\ & -4x_1 + x_2 \leq 0, \\ & x_1, x_2 \geq 0, \\ & x_1 \in \mathbb{Z}, x_2 \in \mathbb{R}. \end{array}$$

[4 pts]

Problem 3.4. Let $\alpha > 0$ be given. Find the minimal description of the convex hull of the set

$$M = \{(x, y) \in \mathbb{Z} \times \mathbb{R} : x - y \leq \alpha, y \geq 0\}.$$

[2 pts]

Problem 3.5. How can we modify the branch-and-bound algorithm to find a "sufficiently good" feasible solution whose objective value is within p% of the optimum value? [2 pts]

Problem 3.6. Tighten the bounds for the integer variables $x_1, \ldots, x_6$ subject to the constraints

$$\begin{array}{l} 2x_1 + 7x_2 - 3x_3 + 6x_4 - 9x_5 + x_6 \leq -12, \\ x_1 - 2x_2 + x_3 + 4x_4 + 2x_5 - 3x_6 \leq 13, \\ x_1 \in [1, 4],\ x_2 \in [0, 7],\ x_3 \in [4, 10],\ x_4 \geq 2,\ x_5 \in [0, 2],\ x_6 \geq 0, \\ x_1, \ldots, x_6 \in \mathbb{Z}. \end{array}$$

[3 pts]