(4) Heuristics, decompositions & column generation

Exercise 4.1. Consider the set $X = \{x \in \{0,1\}^n : \sum_{j=1}^n a_j x_j \leq b\}$ with $a_j \geq 0$ for each $j \in \{1, \ldots, n\}$. Under what conditions

- (a) is the set X empty?
- (b) is the constraint $\sum_{j=1}^{n} a_j x_j \leq b$ redundant?
- (c) is the constraint $x_j = 0$ valid?
- (d) is the constraint $x_i + x_j \leq 1$ valid?

Exercise 4.2. Solve the following 0-1 program by deriving and combining logical inequalities:

$$7x_{1} + 3x_{2} - 4x_{3} - 2x_{4} \le 1$$

$$-2x_{1} + 7x_{2} + 3x_{3} + x_{4} \le 6$$

$$-2x_{2} - 3x_{3} - 6x_{4} \le -5$$

$$3x_{1} - 2x_{3} \ge -1$$

$$x_{1}, \dots, x_{4} \in \{0, 1\}$$

Exercise 4.3. Apply Lagrangian relaxation with the subgradient method to the integer program:

$$\begin{array}{rll} \max & 16x_1 + 10x_2 & + 4x_4 \\ \text{s.t.} & 8x_1 + 2x_2 + x_3 + 4x_4 \leq 10 \\ & x_1 + x_2 & \leq 1 \\ & x_3 + x_4 \leq 1 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{array}$$

Exercise 4.4. Apply the column generation algorithm to the following cutting stock problem:

- company stores rolls of length 218 cm,
- a customer has ordered 44 pieces of length 81 cm, 3 pieces of length 70 cm and 48 pieces of length 68 cm.