(4) Special cases: Knapsack, TSP and others

Exercise 4.1. Solve the following knapsack problem using the branch-and-bound method:

$$\begin{array}{ll} \max & 17x_1 + 10x_2 + 25x_3 + 17x_4 \\ \text{za podm.} & 5x_1 + 3x_2 + 8x_3 + 7x_4 \leq 12, \\ & x_1, x_2, x_3, x_4 \in \{0,1\}. \end{array}$$

Exercise 4.2. Solve the following knapsack problem using the pseudopolynomial algorithm:

$$\begin{array}{ll} \max & x_1 + 5x_2 + 3x_3 + x_4 + 2x_5 \\ \text{s.t.} & 3x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 \leq 7 \\ & x_1, \ldots, x_5 \in \{0,1\} \end{array}$$

Exercise 4.3. Consider the knapsack problem in Exercise 4.2.

(a) Find the optimum of the linear relaxation and use it to construct a minimal cover inequality.

(b) Use lifting to strengthen the cut and solve the relaxation of the new problem.

Exercise 4.4. Apply the following heuristics to the symmetric TSP given by the cost matrix

$$\begin{pmatrix} - & 8 & 4 & 9 & 9 \\ 8 & - & 6 & 7 & 10 \\ 4 & 6 & - & 5 & 6 \\ 9 & 7 & 5 & - & 4 \\ 9 & 10 & 6 & 4 & - \end{pmatrix},$$

(a) nearest neighbor,

(b) greedy algorithm,

(c) savings heuristics,

(d) nearest insertion,

(e) minimum spanning tree method,

(f) 2-change method.

Exercise 4.5. Solve the 1-tree relaxation of the symmetric TSP given by the distance matrix

$$\begin{pmatrix} - & 10 & 2 & 4 & 6 & 2 \\ 10 & - & 9 & 3 & 1 & 3 \\ 2 & 9 & - & 5 & 6 & 1 \\ 4 & 3 & 5 & - & 2 & 5 \\ 6 & 1 & 6 & 2 & - & 3 \\ 2 & 3 & 1 & 5 & 3 & - \end{pmatrix}.$$

Exercise 4.6. Use linear relaxation, greedy algorithm and local improvement to solve an instance of the uncapacitated facility location problem with

$$C' = \begin{pmatrix} 3 & 9 & 2 & 6 \\ 5 & 9 & 7 & 6 \\ 0 & 7 & 6 & 6 \\ 6 & 7 & 4 & 0 \end{pmatrix}, \qquad f = \begin{pmatrix} 3 \\ 2 \\ 3 \\ 3 \end{pmatrix}.$$

Exercise 4.7. Let P denote the convex hull of the feasible set for the uncapacitated facility location problem with $m = 2$, i.e.

$$z_{1j} + z_{2j} = 1 \quad \forall j \in [n], \qquad 0 \leq z_{ij} \leq y_i \quad \forall i \in \{1,2\}, j \in [n], \qquad y_1, y_2 \in \{0,1\},$$

where $[n] = \{1, \ldots, n\}$. Prove that $z_{ij} \leq y_i$ is a facet-defining inequality for P for each $i \in \{1,2\}$ and $j \in [n]$. Can you generalize the proof for any $m \in \mathbb{N}$?