(4) Special cases: Knapsack & TSP

Problem 4.1. Decide which of the following variants of the knapsack problem with $A, a, b, c \geq 0$ can be transformed into one of the other forms $(P1) - (P4)$:

(P1) max $c^T x$ subject to $a^T x \leq b$, $x \in \{0,1\}^n$,

(P2) max $c^T x$ subject to $a^T x \leq b$, $x \geq 0$, $x \in \mathbb{Z}^n$,

(P3) max $c^T x$ subject to $A^T x \leq b$, $x \in \{0,1\}^n$,

(P4) max $c^T x$ subject to $a^T x = b$, $x \in \{0,1\}^n$. [4 pts]

Problem 4.2. Find minimal cover inequalities for the set

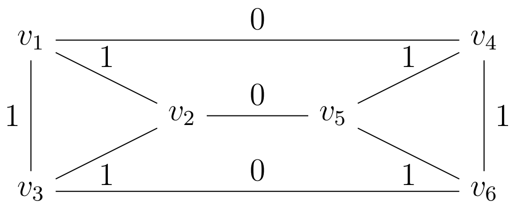
$$\{x \in \{0,1\}^7 : 11x_1 + 6x_2 + 6x_3 + 5x_4 + 5x_5 + 4x_6 + x_7 \leq 19\}.$$ [3 pts]

Problem 4.3. Consider the binary knapsack set

$$K = \{x \in \{0,1\}^n : a^T x \leq b\}$$

with $0 < a_j \leq b$ for all $j \in \{1, \ldots, n\}$. Show that $x_j \geq 0$ defines a facet of $\operatorname{conv}(K)$. [5 pts]

Problem 4.4. Consider a TSP instance on the weighted graph



where the cost of a missing edge is equal to the shortest path between the two nodes. Find a comb inequality cutting off the fractional solution

$$x_{14} = x_{25} = x_{36} = 1,$$
$$x_{12} = x_{23} = x_{13} = x_{46} = x_{56} = x_{45} = \frac{1}{2},$$
(other values x_{ij} are 0). [3 pts]

Problem 4.5. Find all facet-defining inequalities for symmetric TSP with $n = 4$. [5 pts]