(5) Heuristics, decompositions & Software

Problem 5.1. Considering a program

$$\max\ c^T x + d^T y \ \text{ s. t. } Ax + By \le b,\ x, y \ge 0,\ x \in \mathbb{Z}^n, y \in \mathbb{Z}^m,$$

where x denotes the "more important" variables, the relax & fix heuristic approximates the solution in two steps: First, solve the relaxed problem with $x \in \mathbb{Z}^n$, $y \in \mathbb{R}^m$ and denote the optimal solution by (x^*, y^*). Then, fix $x = x^*$ and solve the original program for y to obtain the heuristic solution (x^*, y').

Apply the relax & fix heuristic to compute an approximation of the optimal solution for the integer linear program

$$\begin{array}{ll} \max & 13x + 8y \\ \text{subject to} & x + 2y \le 10, \\ & 5x + 2y \le 20, \\ & x, y \in \mathbb{N}_0. \end{array}$$

[3 pts]

Problem 5.2. Use Langrangian relaxation to solve the integer linear program:

$$\begin{array}{ll} \max & 9x_1 + 4x_2 + 15x_3 \\ \text{subject to} & 3x_1 + 2x_2 + \ \ 4x_3 \le 5, \\ & x_1, x_2, x_3 \in \{0, 1\}. \end{array}$$

Determine the optimal value of the relaxation (P_u) for all possible values $u \ge 0$ and find the best upper bound on the optimal value of the integer program, which can be obtained from these relaxations (i.e. optimum of the Lagrangian relaxation). [4 pts]

Problem 5.3. Apply Lagrangian relaxation to the knapsack problem

$$\begin{array}{ll} \max & c^T x \\ \text{subject to} & a^T x \le b,\ x \in \{0, 1\}^n, \end{array}$$

with $a > 0$, $c > 0$, analyze the resulting program and compare it with the linear-programming relaxation. [4 pts]

For the following problems, formulate an integer linear programming model and find the optimal solution using a modeling/programming language and a solver of your choice. In addition to the obtained optimal solution and optimal value, also turn in your commented source code used for solving the model.

Problem 5.4. Find the chromatic number (i.e. the smallest number of colors needed to color the vertices so that no two adjacent vertices share the same color) and the corresponding coloring of the graph $G = (V, E)$ with $V = \{1, \ldots, 8\}$ and

$$\begin{aligned} E = \{&(1,3), (1,4), (1,5), (1,8), (2,3), (2,6), (2,8), \\ &(3,5), (3,8), (4,6), (5,7), (5,8), (6,8), (7,8)\}. \end{aligned}$$

[3 pts]

Problem 5.5. Solve the symmetric travelling salesman problem with 5 cities and distance matrix

$$(c_e) = \begin{pmatrix} - & 10 & 2 & 4 & 6 \\ - & - & 9 & 3 & 1 \\ - & - & - & 5 & 6 \\ - & - & - & - & 2 \end{pmatrix}.$$

[3 pts]

Problem 5.6. Solve the following Sudoku problem:

4		5						
			7		2	6		
3	6				1		9	
								5
	1					7		
						5		
			9	8		1	2	7
		9	5	4			3	

[3 pts]