2 Interval linear systems of equations

Exercise 1. The following conditions are necessary for regularity of an interval matrix $\boldsymbol{A} \in \mathbb{IR}^{n\times n}$. Decide which of the conditions are also sufficient:

a) The matrix $A_{yz} = A^c + \operatorname{diag}(y)A^\Delta\operatorname{diag}(z)$ is non-singular for each $y, z \in \{\pm 1\}^n$. [2 pts]

b) Each matrix $A \in \mathbb{R}^{n\times n}$ with $a_{ij} \in \{\underline{a}_{ij}, \overline{a}_{ij}\}$ is non-singular. [2 pts]

Exercise 2. Use different conditions to check regularity of the following interval matrices:

$$\boldsymbol{A} = \begin{pmatrix} [-2,0] & [3,5] \\ [-3,-2] & [-4,-2] \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} [-2,1] & [-3,-1] \\ [1,2] & [2,4] \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} [-1,1] & [2,6] \\ [-4,-2] & [-1,1] \end{pmatrix}.$$ [5 pts]

Exercise 3. An interval matrix $\boldsymbol{A} \in \mathbb{IR}^{n\times n}$ is inverse non-negative if $A^{-1} \geq 0$ holds for each $A \in \boldsymbol{A}$. Let $\boldsymbol{A} \in \mathbb{IR}^{n\times n}$ be regular. Prove that $\boldsymbol{A}$ is inverse non-negative if it satisfies the following conditions:

a) $\rho(I_n - \underline{A}) < 1,\ \overline{A} \leq I_n,\ \forall i\colon \boldsymbol{a}_{ii} = 1,$ [3 pts]

b) $\rho(I_n - \underline{A}) < 1,\ \overline{A} \leq I_n,\ \forall i\colon \underline{a}_{ii} \geq 1.$ [3 pts]

Exercise 4. Consider the equation of an electrical circuit in the form

$$\begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}.$$

Given the values

$$V_1 = 10, \quad V_2 = 5, \quad R_1 = R_2 = R_3 = 1000 \pm 10\%,$$

compute an interval enclosure for I_1 and I_2. Can you exploit the parametric dependencies to obtain a tighter enclosure?

Compare different methods for finding the envelopes – implement your own method(s) of choice or utilize the available tools for interval computations (e.g. Versoft, Lime, Intlab, ...). [5 pts]

Exercise 5. Let A_c be a randomly generated 2-by-2 matrix

$$A_c = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

and let $b = (b_1, b_2)^T$ be a randomly generated vector. Create a program to draw the set of all feasible solutions of the interval linear system

$$[A_c - D, A_c + D]x = [b - d, b + d]$$

for $\delta \in \{0.1, 0.2, \ldots, 1.0\}$, where $D \in \mathbb{R}^{2\times 2}$ is a matrix with all entries $D_{ij} = \delta$ and $d \in \mathbb{R}^2$ is a vector with $d_1 = d_2 = \delta$. Can you estimate how the feasible set changes when changing the parameter δ for fixed data A_c and b? [5 pts]