3 Interval linear algebra

Exercise 1. Describe and illustrate the set of all weak solutions and the set of all strong solutions to the interval linear system:

$$\begin{pmatrix} [-3,1] & [1,2] \\ [-1,1] & -2 \\ [1,2] & [-2,1] \\ -2 & [-1,1] \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} [3,4] \\ [3,4] \\ [3,4] \\ [3,4] \end{pmatrix}.$$

[4 pts]

Exercise 2. Compute an interval enclosure for eigenvalues $\boldsymbol{\lambda}_i(\boldsymbol{A}^S)$ of the interval matrix

$$\begin{pmatrix} -3 & [4,6] & [-3,-1] \\ [4,6] & [-5,-1] & [1,3] \\ [-3,-1] & [1,3] & -1 \end{pmatrix}.$$

[3 pts]

Exercise 3. Let $c \in \mathbb{R}$ be fixed and let $\boldsymbol{A} \in \mathbb{IR}^{n\times n}$. Prove that checking whether $c \in \lambda_i(\boldsymbol{A}^S)$ for some $i \in \{1,\dots,n\}$ is an NP-hard problem. [3 pts]

Exercise 4. Given an interval vector $\boldsymbol{v} \in \mathbb{IR}^n$, find intervals of eigenvalues $\boldsymbol{\lambda}_i(\boldsymbol{A}^S)$, $i \in \{1,\dots,n\}$, for the diagonal matrix $\boldsymbol{A}^S = \operatorname{diag}(\boldsymbol{v})$. [4 pts]

Exercise 5. Decide whether the following propositions are true or false:

a) If the matrix $\boldsymbol{A}^S$ is positive semidefinite, then $\lambda_n(A_c) \geq \rho(A_\Delta)$. [2 pts]

b) If the matrix $\boldsymbol{A}^S$ is positive definite, then $\lambda_n(A_c) > \rho(A_\Delta)$. [2 pts]

Exercise 6. Are the interval matrices $\boldsymbol{A}^S$, $\boldsymbol{B}^S$ positive (semi-)definite?

$$\text{a) } \boldsymbol{A} = \begin{pmatrix} [6,8] & [-3,-1] & [-2,0] \\ [-3,-1] & [7,9] & [1,3] \\ [-2,0] & [1,3] & [4,6] \end{pmatrix}, \qquad \text{b) } \boldsymbol{B} = \begin{pmatrix} [13,15] & [3,7] & [9,11] \\ [3,7] & [4,8] & 5 \\ [9,11] & 5 & [13,15] \end{pmatrix}.$$

[4 pts]

Exercise 7. Characterize some classes of matrices for which positive (semi)definiteness can be checked easily and formulate the corresponding conditions for testing the property. [3 pts]