Solving and visualizing nonlinear constraint satisfaction problems

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Introduction

The concept of constraint satisfaction can be used to formalize many practical problems involving a set of variables with given properties. Our talk will focus on continuous constraint satisfaction problems with interval domains, where the solution set is described by a system of nonlinear inequalities. We would like to introduce an interval solver for nonlinear constraints [1] with a visualization software, which will be included in the interval toolbox Lime.

Interval solver for nonlinear constraints

Our solver is based on the branch-and-bound algorithm Sivia (Set Inversion via Interval Analysis, see [2]), which approximates the solution set $\mathbb{X}$ of a nonlinear problem using 3 sets $\mathcal{S}, \mathcal{N}, \mathcal{E}$ of non-overlapping interval boxes (also referred to as a paving) satisfying $\mathcal{S} \subseteq \mathbb{X} \subseteq (\mathcal{S} \cup \mathcal{E})$ and $\mathbb{X} \cap \mathcal{N} = \emptyset$. Interval contractors, such as the forward-backward contractor, are used to improve the efficiency of the basic algorithm. We have also implemented a simple unification procedure to decrease the number of boxes characterizing the solution set of a given problem.

An interesting application of the algorithm is the visualization of complex interval arithmetic. If we define (rectangular) complex intervals as two-dimensional interval boxes [3], we can describe an exact

product (or quotient) of two complex intervals using nonlinear constraints and variables with continuous domains.

Interval data visualization

The visualization software implements several methods for approximating the exact solution set of a nonlinear system using the results obtained by the solver. The approximation methods divide the set of undecided boxes in the resulting paving into the satisfying set and the non-satisfying set.

The visualization software communicates with the user through a simple and user-friendly graphical interface. Two or more pavings can be displayed at once for a visual comparison of the sets. It also offers a wide range of minor functions, which can be used to customize the visualization settings.

References

[**1**] E. Garajová, *Intervalový solver nelineárních podmínek [Interval solver for nonlinear constraints]*, Bachelor's thesis, 2014.

[**2**] L. Jaulin et al., *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control and Robotics*, Springer-Verlag, 2001.

[**3**] R. E. Boche, *Complex interval arithmetic with some applications*, Technical Report LMSC4-22-66-1, Lockheed Missiles and Space Co., Sunnyvale, CA, USA, 1966.