Solving and visualizing nonlinear constraint satisfaction problems

Elif Garajová, Martin Mečiar

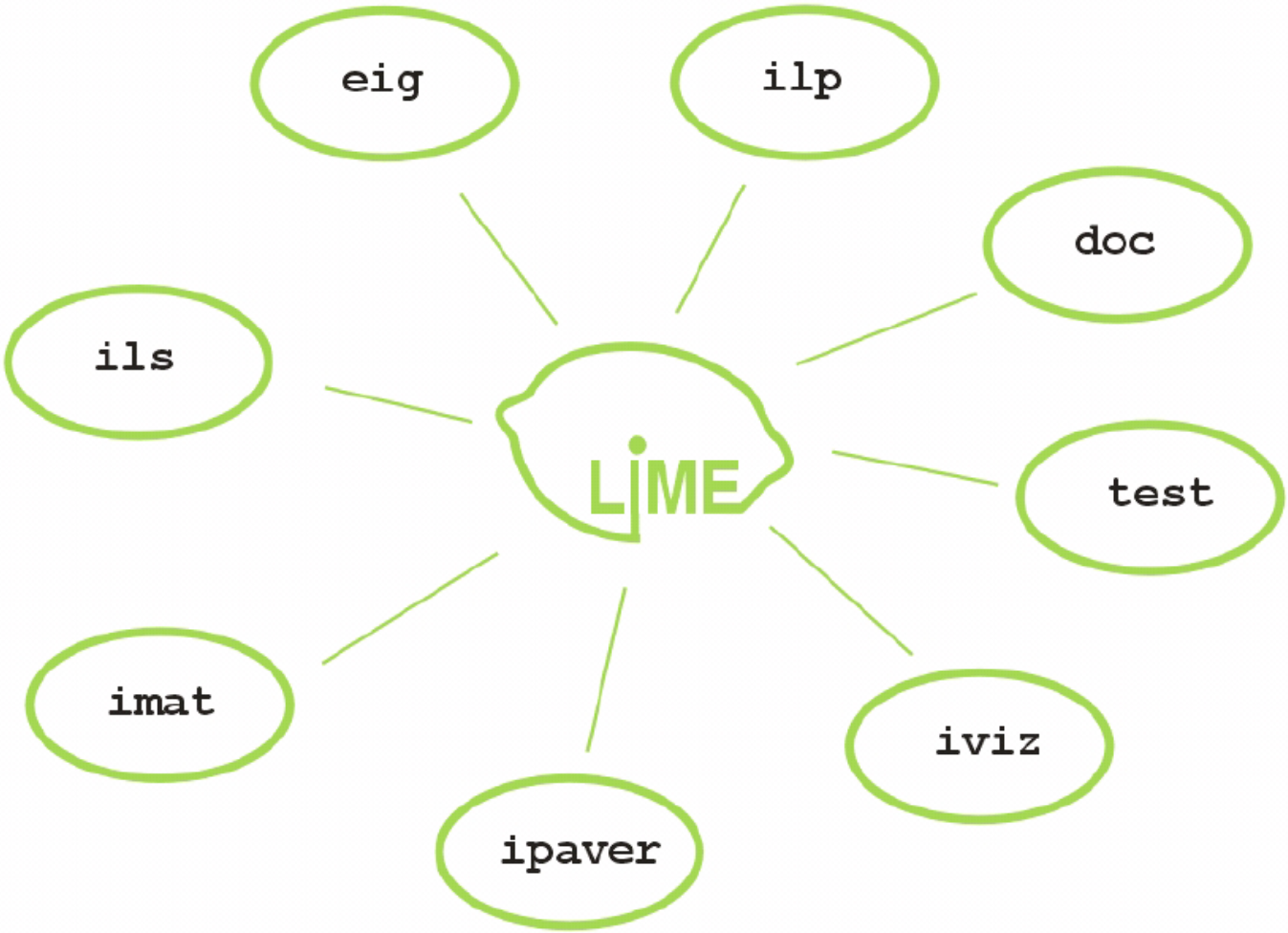
Department of Applied Mathematics
Faculty of Mathematics and Physics
Charles University in Prague

SWIM, June 2015

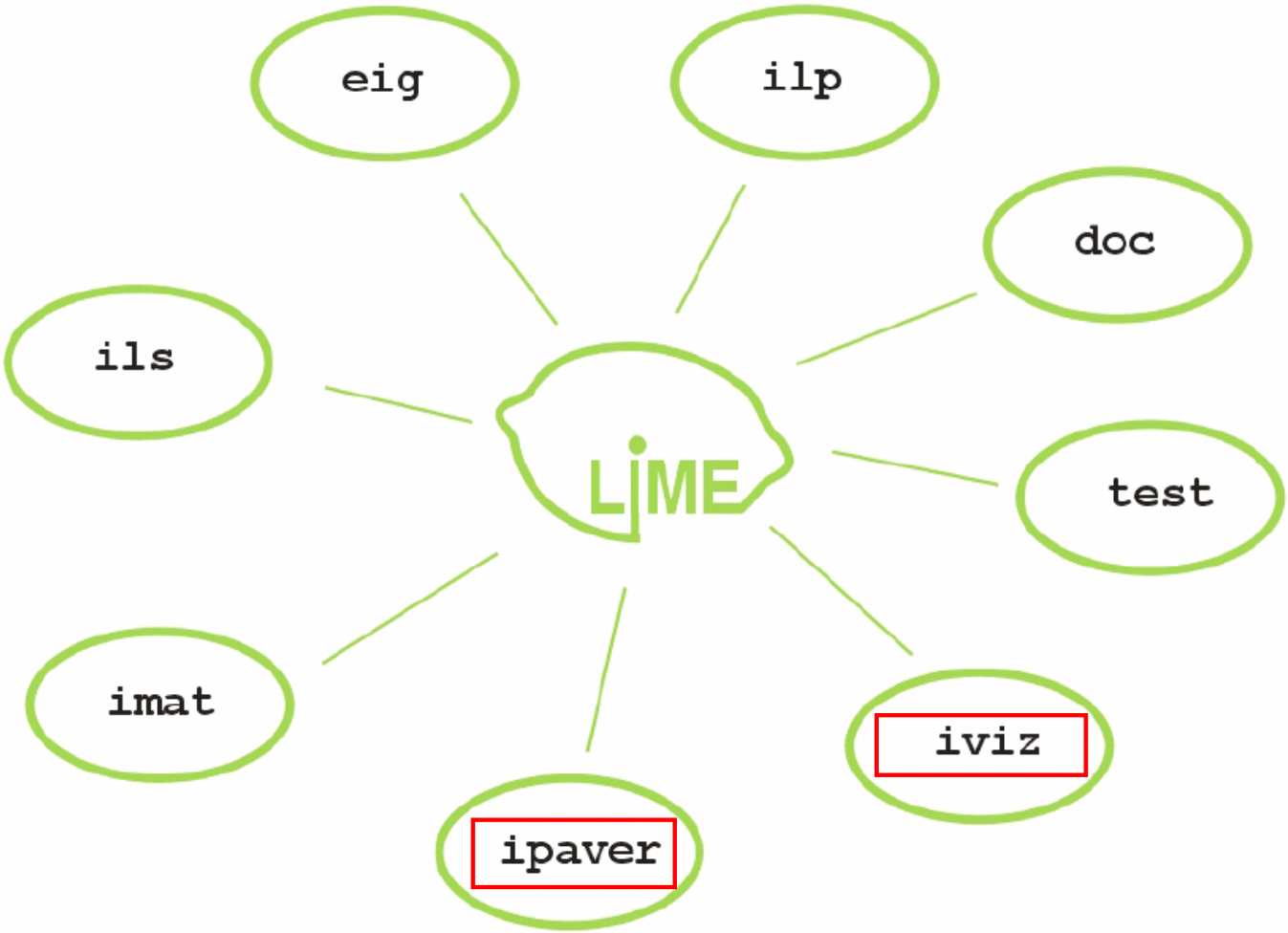
Outline

1. Interval solver for nonlinear constraints
2. Application: Complex intervals
3. Visualization techniques

Library of Interval MEthods



Library of Interval MEthods



Problem: Solving a nonlinear CSP

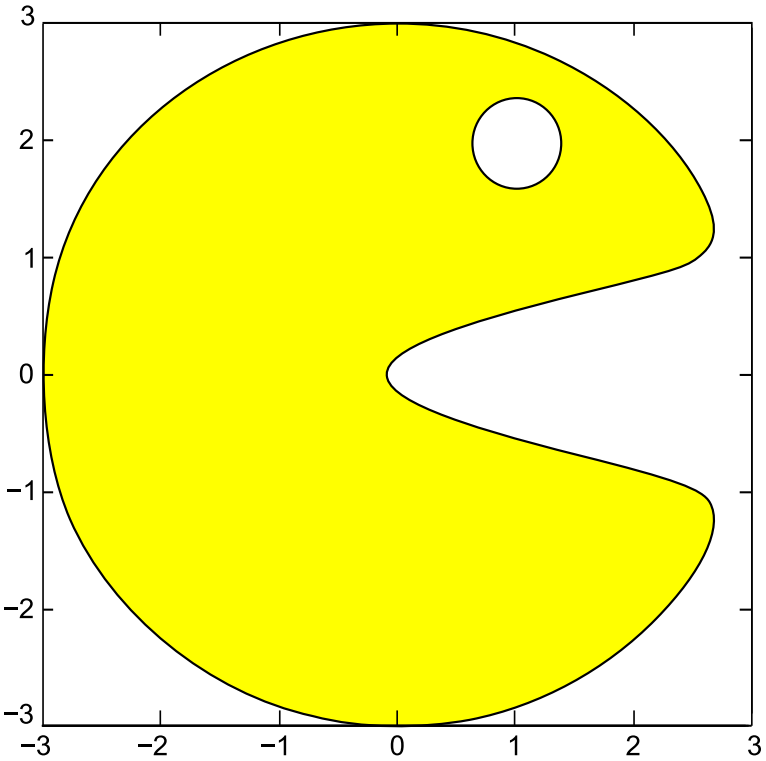
Find the set of all $(x, y) \in [-3, 3] \times [-3, 3]$ satisfying:

$$\left(x^2 + y^2 - 9\right)\left(\frac{1}{3}x - y^2\right) \geq \frac{1}{2}$$

$$(y-2)^2 + (x-1)^2 \geq \frac{1}{7}$$

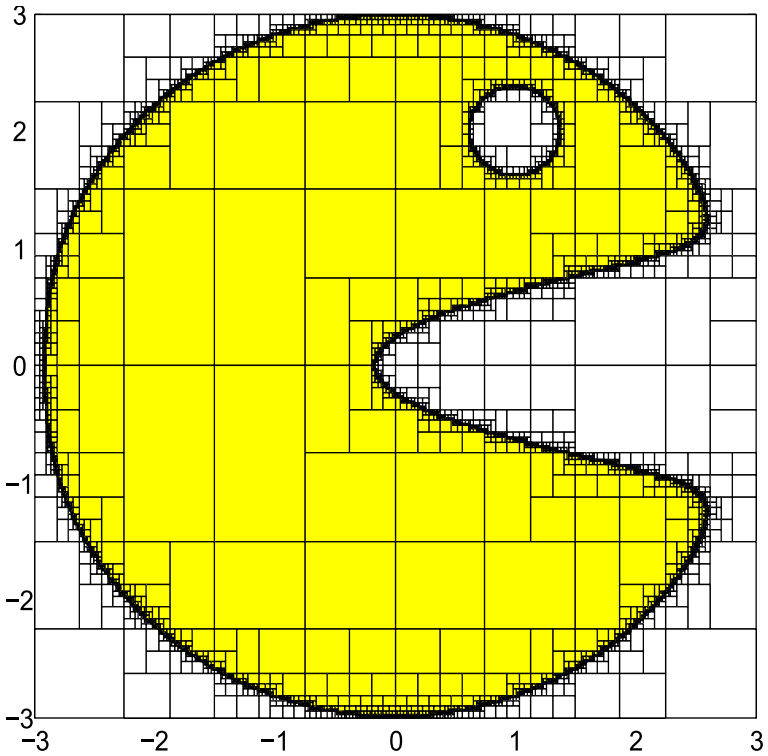
- How to describe the solution set?
- How to find all solutions?

Describing the solution set



- visual representation of the set
- projection from higher dimensions
- basic information about the set

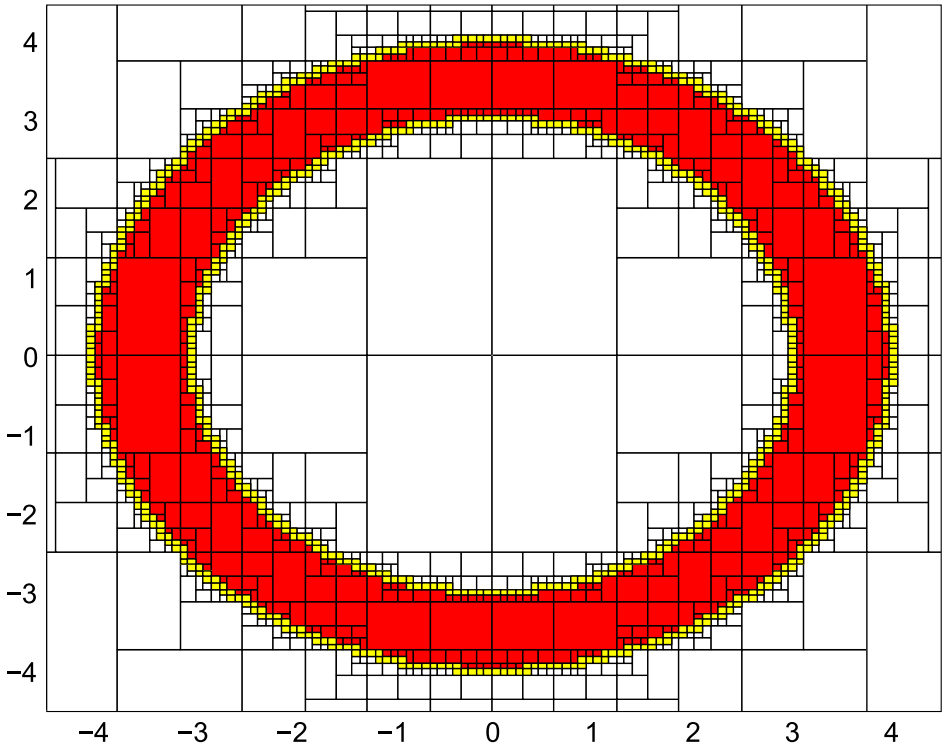
Describing the solution set



- visual representation of the set
- projection from higher dimensions
- basic information about the set

- description using interval boxes
- outer and inner approximation

Solution: SIVIA



inner approximation: $\mathcal{S} \subset \mathbb{X}$

outer approximation: $\mathbb{X} \subset (\mathcal{S} \cup \mathcal{E})$

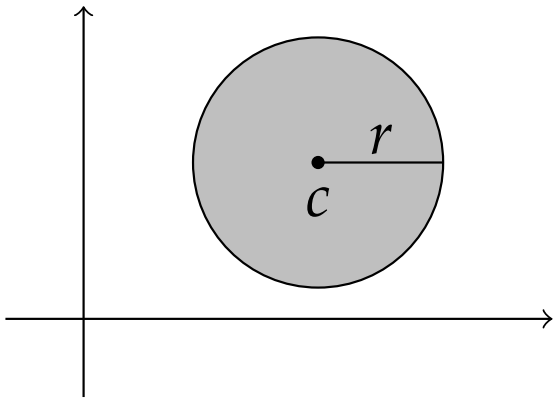
boxes with no solutions: $(\mathcal{N} \cap \mathbb{X}) = \emptyset$

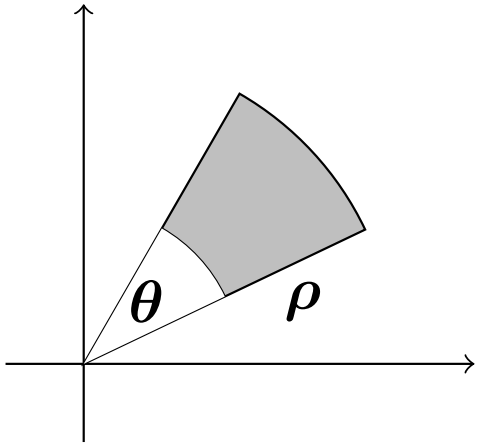
Interval solver for nonlinear constraints

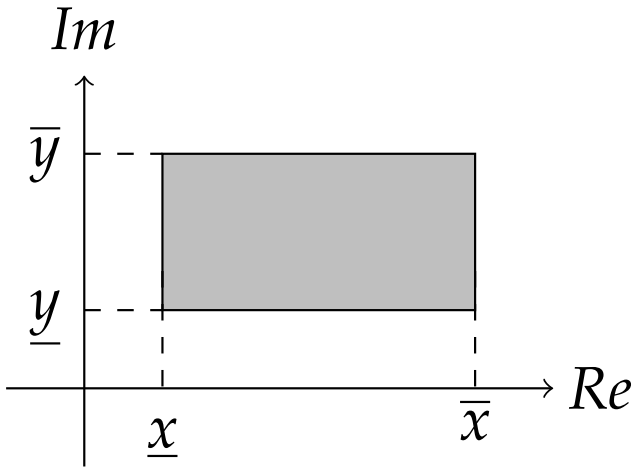
- solver based on the SIVIA algorithm
- uses interval contractors to enhance its efficiency
- written in MATLAB (and C++) using the INTLAB toolbox

- the interval solver can:
  - solve a nonlinear CSP using interval methods
  - reduce the number of boxes on the output
  - plot the solution set (or its projection) in 2D
  - visualize complex interval arithmetic

Complex intervals







Complex interval arithmetic I

Exact operations

$$(\mathbf{a}, \mathbf{b}) + (\mathbf{c}, \mathbf{d}) = (\mathbf{a} + \mathbf{c}, \mathbf{b} + \mathbf{d})$$
$$(\mathbf{a}, \mathbf{b}) - (\mathbf{c}, \mathbf{d}) = (\mathbf{a} - \mathbf{c}, \mathbf{b} - \mathbf{d})$$

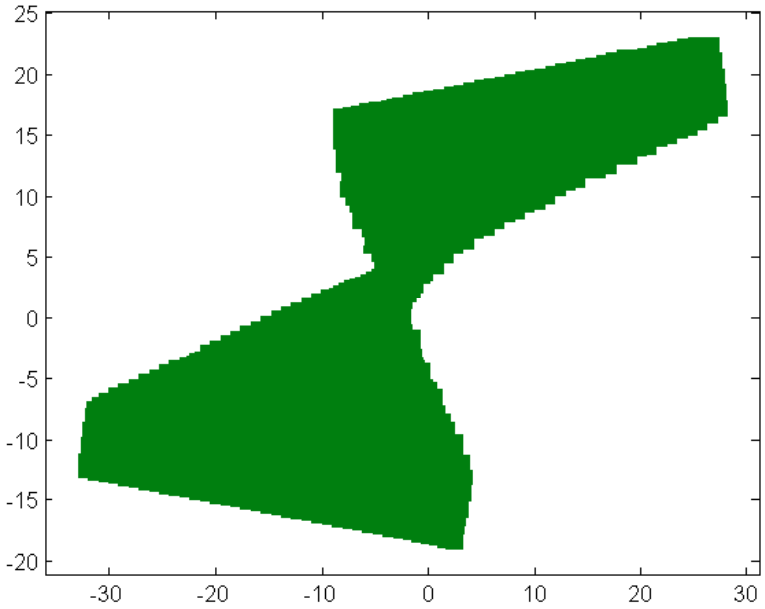
Overestimated operations

$$(\mathbf{a}, \mathbf{b}) \cdot (\mathbf{c}, \mathbf{d}) = (\mathbf{ac} - \mathbf{bd}, \mathbf{ad} + \mathbf{bc})$$
$$\frac{(\mathbf{a}, \mathbf{b})}{(\mathbf{c}, \mathbf{d})} = \left(\frac{\mathbf{ac} + \mathbf{bd}}{\mathbf{c}^2 + \mathbf{d}^2}, \frac{\mathbf{bc} - \mathbf{ad}}{\mathbf{c}^2 + \mathbf{d}^2} \right)$$

Complex interval arithmetic II

Interval operation: $(\mathbf{a}, \mathbf{b}) \cdot (\mathbf{c}, \mathbf{d}) = (\mathbf{ac} - \mathbf{bd}, \mathbf{ad} + \mathbf{bc})$

Exact operation: $\{(a + bi) \cdot (c + di) \mid (a, b) \in (\mathbf{a}, \mathbf{b}), (c, d) \in (\mathbf{c}, \mathbf{d})\}$



Visualizing nonlinear CSPs